TWO-WAY RADAR EQUATION (MONOSTATIC)

In this section the radar equation is derived from the one-way equation (transmitter to receiver) which is then extended to the two-way radar equation. The following is a summary of the important equations to be derived here:

TWO-WAY RADAR EQUATION (MONOSTATIC)

Peak power at the radar receiver input is:

$$P_r = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4} = P_t G_t G_r \left[\frac{\sigma c^2}{(4\pi)^3 f^2 R^4}\right]^*$$

Note: $\lambda = c/f$ *and* σ = *RCS*
∗*keep* λ *or* c, σ, *and* R *in the same units*

On reducing the above equation to log form we have:

$$10\log P_r = 10\log P_t + 10\log G_t + 10\log G_r + 10\log \sigma - 20\log f - 40\log R - 30\log 4\pi + 20\log c$$

or in simplified terms: $10\log P_r = 10\log P_t + 10\log G_t + 10\log G_r + G_\sigma - 2\alpha_1$ (in dB)

Note: Losses due to antenna polarization and atmospheric absorption (Sections 3-2 and 5-1) are not included in these equations.

Target gain factor, $G_\sigma = 10\log \sigma + 20\log f_1 + K_2$ (in dB)

K_2 Values (dB)	RCS (σ) (units)	f_1 in MHz K_2 =	f_1 in GHz K_2 =
	m^2	-38.54	21.46
	ft^2	-48.86	11.14

One-way free space loss, $\alpha_1 = 20\log(f_1 R) + K_1$ (in dB)

K_1 Values (dB)	Range (units)	f_1 in MHz K_1 =	f_1 in GHz K_1 =
	NM	37.8	97.8
	Km	32.45	92.45
	m	-27.55	32.45
	yd	-28.33	31.67
	ft	-37.87	22.13

Figure 1 illustrates the physical concept and equivalent circuit for a target being illuminated by a monostatic radar (transmitter and receiver co-located). Note the similarity of Figure 1 to Figure 3 in Section 4-3. Transmitted power, transmitting and receiving antenna gains, and the one-way free space loss are the same as those described in Section 4-3. The physical arrangement of the elements is different, of course, but otherwise the only difference is the addition of the equivalent gain of the target RCS factor.

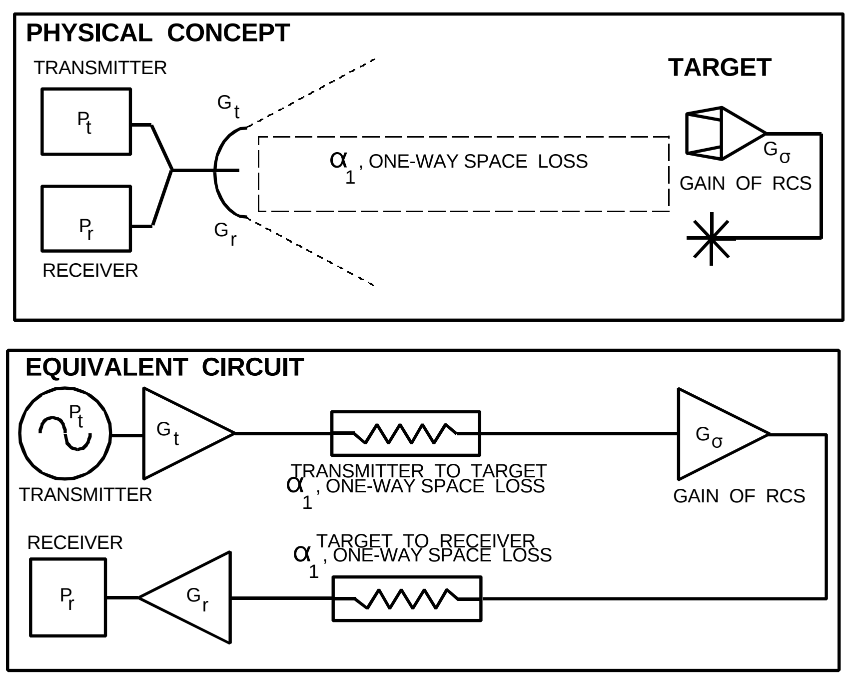

Figure 1. The Two-Way Monostatic Radar Equation Visualized

From Section 4-3, One-Way Radar Equation / RF Propagation, the power in the receiver is:

$$\begin{matrix} Received\ Signal \\ at\ Target \end{matrix} = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2} \qquad \textbf{[1]}$$

From equation [3] in Section 4-3:

$$Antenna\ Gain, G = \frac{4\pi A_e}{\lambda^2} \qquad \textbf{[2]}$$

Similar to a receiving antenna, a radar target also intercepts a portion of the power, but reflects (reradiates) it in the direction of the radar. The amount of power reflected toward the radar is determined by the Radar Cross Section (RCS) of the target. RCS is a characteristic of the target that represents its size as seen by the radar and has the dimensions of area (σ) as shown in Section 4-11. RCS area is not the same as physical area. But, for a radar target, the power reflected in the radar's direction is equivalent to re-radiation of the power captured by an antenna of area σ (the RCS). Therefore, the effective capture area (A_e) of the receiving antenna is replaced by the RCS (σ).

$$G_r = \frac{4\pi\sigma}{\lambda^2} \qquad \textbf{[3]}$$

so we now have:

$$\begin{matrix} Reflected\ Signal \\ from\ target \end{matrix} = \frac{P_t G_t \lambda^2 4\pi\sigma}{(4\pi R)^2 \lambda^2} \qquad \textbf{[4]}$$

The equation for the power reflected in the radar's direction is the same as equation [1] except that $P_t\ G_t$, which was the original transmitted power, is replaced with the reflected signal power from the target, from equation [4]. This gives:

$$\begin{matrix} Reflected\ Signal\ Received\ Back \\ at\ Input\ to\ Radar\ Receiver \end{matrix} = \frac{P_t G_t \lambda^2 4\pi\sigma}{(4\pi R)^2 \lambda^2}\ x\ \frac{G_r \lambda^2}{(4\pi R)^2} \qquad \textbf{[5]}$$

S ⬇ **12 dB** *(1/16 pwr)*	***TWO WAY SIGNAL STRENGTH (S)*** *__S__ decreases by __12 dB__ when the distance doubles*	**2R** ⬆ **R**
12 dB *(16x pwr)* ⬆ **S**	*__S__ increases by __12 dB__ when the distance is half*	**R** ⬇ **0.5 R**

If like terms are cancelled, the two-way radar equation results. The peak power at the radar receiver input is:

$$P_r = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4} = P_t G_t G_r \left[\frac{\sigma c^2}{(4\pi)^3 f^2 R^4}\right]^{*} \qquad \textbf{[6]}$$

* Note: λ=c/*f* and σ = RCS. Keep λ or c, σ, and R in the same units.

On reducing equation [6] to log form we have:

$$10\log P_r = 10\log P_t + 10\log G_t + 10\log G_r + 10\log \sigma - 20\log f - 40\log R - 30\log 4\pi + 20\log c \qquad \textbf{[7]}$$

<u>Target Gain Factor</u>

If Equation [5] terms are rearranged instead of cancelled, a recognizable form results:

$$S\ (or\ P_r) = (P_t G_t G_r) \cdot \left[\frac{\lambda^2}{(4\pi R)^2}\right] \cdot \left[\frac{4\pi\sigma}{\lambda^2}\right] \cdot \left[\frac{\lambda^2}{(4\pi R)^2}\right] \qquad \textbf{[8]}$$

In log form:

$$10\log[S\ (or\ P_r)] = 10\log P_t + 10\log G_t + 10\log G_r + 20\log\left[\frac{\lambda}{4\pi R}\right] + 10\log\left[\frac{4\pi\sigma}{\lambda^2}\right] + 20\log\left[\frac{\lambda}{4\pi R}\right] \qquad \textbf{[9]}$$

The fourth and sixth terms can each be recognized as -α, where α is the one-way free space loss factor defined in Section 4-3. The fifth term containing RCS (σ) is the only new factor, and it is the "Target Gain Factor".

In simplified terms the equation becomes:

$$10\log [S \text{ (or } P_r)] = 10\log P_t + 10\log G_t + 10\log G_r + G_\sigma - 2\alpha_1 \quad \text{(in dB)} \qquad \textbf{[10]}$$

Where α_1 and G_σ are as follows:

From Section 4-3, equation [11], the space loss in dB is given by:

$$\alpha_1 = 20\log\left[\frac{4\pi f R}{c}\right]^{*} = 20\log f_1 R + K_1 \quad \textit{where } K_1 = 20\log\left[\frac{4\pi}{c}\cdot(\textit{Conversion units if not in m/}\text{sec}, \textit{ m, and Hz})\right] \qquad \textbf{[11]}$$

* Keep c and R in the same units. The table of values for K_1 is again presented here for completeness. The constant, K_1, in the table includes a range and frequency unit conversion factor.

One-way free space loss, $\alpha_1 = 20\log(f_1 R) + K_1$ (in dB)			
K_1 Values (dB)	Range (units)	f_1 in MHz K_1 =	f_1 in GHz K_1 =
	NM	37.8	97.8
	Km	32.45	92.45
	m	-27.55	32.45
	yd	-28.33	31.67
	ft	-37.87	22.13

While it's understood that RCS is the antenna aperture area equivalent to an isotropically radiated target return signal, the target gain factor represents a gain, as shown in the equivalent circuit of Figure 1. The Target Gain Factor expressed in dB is G_σ as shown in equation [12].

$$G_\sigma = 10\log\left[\frac{4\pi\sigma}{\lambda^2}\right] = 10\log\left[\frac{4\pi\sigma f^2}{c^2}\right] = 10\log\sigma + 20\log f_1 + K_2 \quad (\textit{in dB}) \qquad \textbf{[12]}$$

$$\textit{where}: K_2 = 10\log\left[\frac{4\pi}{c^2}\cdot\left(\begin{matrix}\textit{Frequency and RCS}\\ \textit{conversions as required}\end{matrix}\ \frac{(\textit{Hz to MHz or GHz})^2}{(\textit{meters to feet})^2}\right)\right]$$

The "Target Gain Factor" (G_σ) is a composite of RCS, frequency, and dimension conversion factors and is called by various names: "Gain of RCS", "Equivalent Gain of RCS", "Gain of Target Cross Section", and in dB form "Gain-sub-Sigma".

If frequency is given in MHz and RCS (σ) is in m^2, the formula for G_σ is:

$$G_\sigma = 10\log\sigma + 20\log f_1 + 10\log\left[4\pi\cdot\left(\frac{\text{sec}}{3x10^8 m}\right)^2\cdot m^2\cdot\left(\frac{1x10^6}{\text{sec}}\right)^2\right] \qquad \textbf{[13]}$$

or:

$$G_\sigma = 10\log\sigma + 20\log f_1 - 38.54 \quad (\textit{in dB}) \qquad \textbf{[14]}$$

For this example, the constant K_2 is -38.54 dB. This value of K_2 plus K_2 for other area units and frequency multiplier values are summarized in the adjoining table.

Target gain factor, $G_\sigma = 10\log\sigma + 20\log f_1 + K_2$ (in dB)			
K_2 Values (dB)	RCS (σ) (units)	f_1 in MHz K_2 =	f_1 in GHz K_2 =
	m^2	-38.54	21.46
	ft^2	-48.86	11.14

In the two-way radar equation, the one-way free space loss factor (α_1) is used twice, once for the radar transmitter to target path and once for the target to radar receiver path. The radar illustrated in Figure 1 is monostatic so the two path losses are the same and the values of the two α_1's are the same.

If the transmission loss in Figure 1 from P_t to G_t equals the loss from G_r to P_r , and $G_r = G_t$, then equation [10] can be written as:

$$10\log\,[S \text{ or } P_r] = 10\log P_t + 20\log G_{tr} - 2\alpha_1 + G_\sigma \quad \text{(in dB)} \qquad \textbf{[15]}$$

The space loss factor (α_1) and the target gain factor (G_σ) include all the necessary unit conversions so that they can be used directly with the most common units. Because the factors are given in dB form, they are more convenient to use and allow calculation without a calculator when the factors are read from a chart or nomograph.

Most radars are monostatic. That is, the radar transmitting and receiving antennas are literally the same antenna. There are some radars that are considered "monostatic" but have separate transmitting and receiving antennas that are co-located. In that case, equation [10] could require two different antenna gain factors as originally derived:

$$10\log\,[S \text{ or } P_r] = 10\log P_t + 10\log G_t + 10\log G_r - 2\alpha_1 + G_\sigma \quad \text{(in dB)} \qquad \textbf{[16]}$$

Note: To avoid having to include additional terms for these calculations, always combine any transmission line loss with antenna gain.

Figure 2 is the visualization of the path losses occurring with the two-way radar equation. **Note:** to avoid having to include additional terms, always combine any transmission line loss with antenna gain. Losses due to antenna polarization and atmospheric absorption also need to be included.

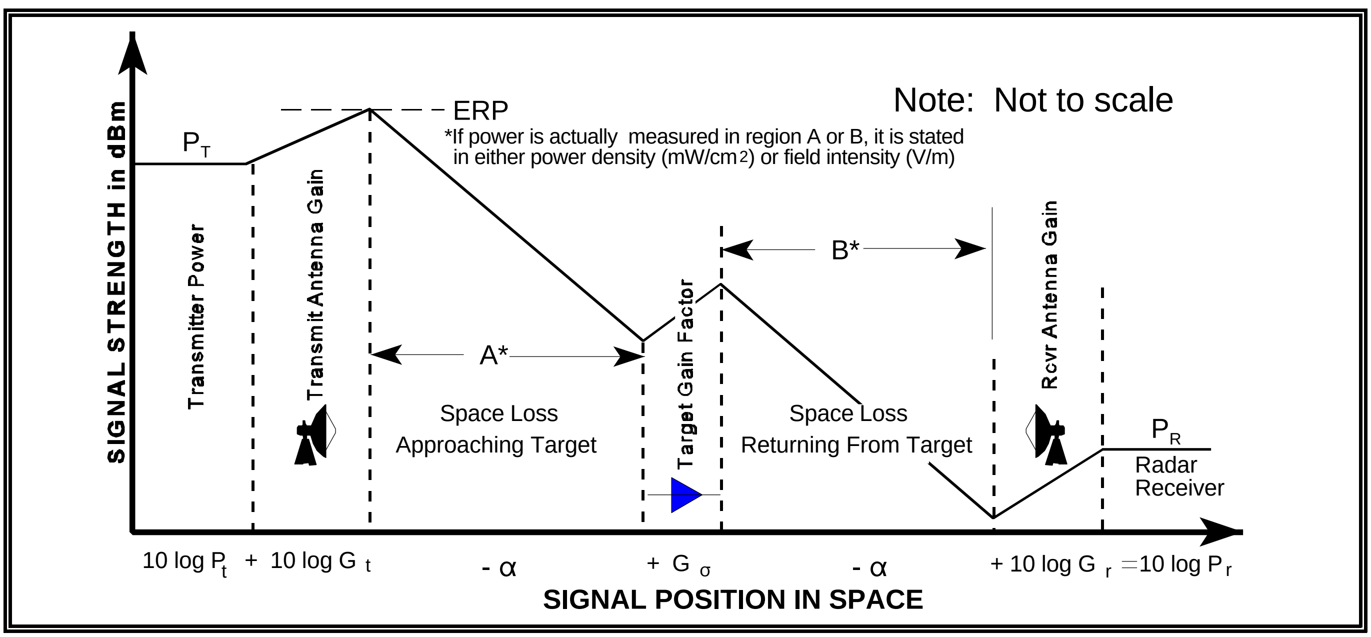

Figure 2. Visualization of Two-Way Radar Equation

RADAR RANGE EQUATION (Two-Way Equation)

The Radar Equation is often called the "Radar Range Equation". The Radar Range Equation is simply the Radar Equation rewritten to solve for maximum Range. The maximum radar range (R_{max}) is the distance beyond which the target can no longer be detected and correctly processed. It occurs when the received echo signal just equals S_{min} .

The Radar Range Equation is then:
$$R_{\max} \cong \left[\frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 S_{\min}}\right]^{\frac{1}{4}} \quad or \quad \left[\frac{P_t G_t G_r c^2 \sigma]}{(4\pi)^3 f^2 S_{\min}}\right]^{\frac{1}{4}} \quad or \quad \left[\frac{P_t G_t A_e \sigma}{(4\pi)^2 S_{\min}}\right]^{\frac{1}{4}} \qquad \textbf{[17]}$$

The first equation, of the three above, is given in Log form by:
$$40\log R_{max} \cong 10\log P_t + 10\log G_t + 10\log G_r + 10\log \sigma - 10\log S_{min} - 20\log f - 30\log 4\pi + 20\log c \qquad \textbf{[18]}$$

As shown previously, Since $K_1 = 20\log [(4\pi/c)$ times conversion units if not in m/sec, m, and Hz], we have:

$$10\log R_{max} \cong ¼\,[10\log P_t + 10\log G_t + 10\log G_r + 10\log \sigma - 10\log S_{min} - 20\log f_1 - K_1 - 10.99 \text{ dB}] \qquad \textbf{[19]}$$

If you want to convert back from dB, then $R_{max} \cong 10^{\frac{MdB}{40}}$

Where M dB is the resulting number within the brackets of equation 19.

One-way free space loss, $\alpha_1 = 20\log (f_1 R) + K_1$ (in dB)			
K_1 Values (dB)	Range (units)	f_1 in MHz K_1 =	f_1 in GHz K_1 =
	NM	37.8	97.8
	Km	32.45	92.45
	m	-27.55	32.45
	yd	-28.33	31.67
	ft	-37.87	22.13

From Section 5-2, Receiver Sensitivity / Noise, S_{min} is related to the noise factors by: $S_{\min} = (S/N)_{\min}(NF)kT_0B$ **[20]**

The Radar Range Equation for a tracking radar (target continuously in the antenna beam) becomes:
$$R_{\max} \cong \left[\frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 (S/N)_{\min}(NF)kT_0B}\right]^{\frac{1}{4}} \quad or \quad \left[\frac{P_t G_t G_r c^2 \sigma}{(4\pi)^3 f^2 (S/N)_{\min}(NF)kT_oB}\right]^{\frac{1}{4}} \quad or \quad \left[\frac{P_t G_t A_e \sigma}{(4\pi)^2 (S/N)_{\min}(NF)kT_oB}\right]^{\frac{1}{4}} \qquad \textbf{[21]}$$

P_t in equations [17], [19], and [21] is the peak power of a CW or pulse signal. For pulse signals these equations assume the radar pulse is square. If not, there is less power since P_t is actually the average power within the pulse width of the radar signal. Equations [17] and [19] relate the maximum detection range to S_{min} , the minimum signal which can be detected and processed (the receiver sensitivity). The bandwidth (B) in equations [20] and [21] is directly related to S_{min}. B is approximately equal to 1/PW. Thus a wider pulse width means a narrower receiver bandwidth which lowers S_{min} , assuming no integration.

One cannot arbitrarily change the receiver bandwidth, since it has to match the transmitted signal. The "widest pulse width" occurs when the signal approaches a CW signal (see Section 2-11). A CW signal requires a very narrow bandwidth (approximately 100 Hz). Therefore, receiver noise is very low and good sensitivity results (see Section 5-2). If the radar pulse is narrow, the receiver filter bandwidth must be increased for a match (see Section 5-2), i.e. a 1 µs pulse requires a bandwidth of approximately 1 MHz. This increases receiver noise and decreases sensitivity.

If the radar transmitter can increase its PRF (decreasing PRI) and its receiver performs integration over time, an increase in PRF can permit the receiver to "pull" coherent signals out of the noise thus reducing S/N_{min} thereby increasing the detection range. Note that a PRF increase may limit the maximum range due to the creation of overlapping return echoes (see Section 2-10).

There are also other factors that limit the maximum practical detection range. With a scanning radar, there is loss if the receiver integration time exceeds the radar's time on target. Many radars would be range limited by line-of-sight/radar horizon (see Section 2-9) well before a typical target faded below S_{min}. Range can also be reduced by losses due to antenna polarization and atmospheric absorption (see Sections 3-2 and 5-1).

Two-Way Radar Equation (Example)

Assume that a 5 GHz radar has a 70 dBm (10 kilowatt) signal fed through a 5 dB loss transmission line to a transmit/receive antenna that has 45 dB gain. An aircraft that is flying 31 km from the radar has an RCS of 9 m^2. What is the signal level at the input to the radar receiver? (There is an additional loss due to any antenna polarization mismatch but that loss will not be addressed in this problem). This problem continues in Sections 4-3, 4-7, and 4-10.

Answer:
Starting with: $10\log S = 10\log P_t + 10\log G_t + 10\log G_r + G_\sigma - 2\alpha_1$ (in dB)

We know that: $\alpha_1 = 20\log f R + K_1 = 20\log (5x31) + 92.44 = 136.25$ dB

and that: $G_\sigma = 10\log \sigma + 20\log f_1 + K_2 = 10\log 9 + 20\log 5 + 21.46 = 44.98$ dB (see Table 1)
(Note: The aircraft transmission line losses (-5 dB) will be combined with the antenna gain (45 dB) for both receive and transmit paths of the radar)

So, substituting in we have: $10\log S = 70 + 40 + 40 + 44.98 - 2(136.25) =$ **-77.52 dBm** @ 5 GHz

The answer changes to -80.44 dBm if the tracking radar operates at 7 GHz provided the antenna gains and the aircraft RCS are the same at both frequencies.

$\alpha_1 = 20\log (7x31) + 92.44 = 139.17$ dB, $G_\sigma = 10\log 9 + 20\log 7 + 21.46 = 47.9$ dB (see Table 1)

$10\log S = 70 + 40 + 40 + 47.9 - 2(139.17) =$ **-80.44 dBm** @ 7 GHz

Table 1. Values of the Target Gain Factor (G_σ) in dB for Various Values of Frequency and RCS

Frequency (GHz)	RCS - Square meters						
	0.05	5	9	10	100	1,000	10,000
0.5 GHz	2.44	22.42	24.98	25.44	35.44	45.44	55.44
1 GHz	8.46	28.46	31.0	31.46	41.46	51.46	61.46
5 GHz	22.44	42.44	44.98	45.44	55.44	65.44	75.44
7 GHz	25.36	45.36	47.9	48.36	58.36	68.36	78.36
10 GHz	28.46	48.46	51.0	51.46	61.46	71.46	81.46
20 GHz	34.48	54.48	57.02	57.48	67.48	77.48	87.48
40 GHz	40.50	60.48	63.04	63.5	73.5	83.5	93.5

Note: Shaded values were used in the examples.

TWO-WAY RADAR RANGE INCREASE AS A RESULT OF A SENSITIVITY INCREASE

As shown in equation [17] $S_{min}^{-1} \propto R_{max}^4$ Therefore, $-10 \log S_{min} \propto 40 \log R_{max}$ and the table below results:

% Range Increase: Range + (% Range Increase) x Range = New Range
i.e., for a 12 dB sensitivity increase, 500 miles +100% x 500 miles = 1,000 miles
Range Multiplier: Range x Range Multiplier = New Range i.e., for a 12 dB sensitivity increase 500 miles x 2 = 1,000 miles

Table 2. Effects of Sensitivity Increase

dB Sensitivity Increase	% Range Increase	Range Multiplier	dB Sensitivity Increase	% Range Increase	Range Multiplier
+ 0.5	3	1.03	10	78	1.78
1.0	6	1.06	11	88	1.88
1.5	9	1.09	12	100	2.00
2	12	1.12	13	111	2.11
3	19	1.19	14	124	2.24
4	26	1.26	15	137	2.37
5	33	1.33	16	151	2.51
6	41	1.41	17	166	2.66
7	50	1.50	18	182	2.82
8	58	1.58	19	198	2.98
9	68	1.68	20	216	3.16

TWO-WAY RADAR RANGE DECREASE AS A RESULT OF A SENSITIVITY DECREASE

As shown in equation [17] $S_{min}^{-1} \propto R_{max}^4$ Therefore, $-10 \log S_{min} \propto 40 \log R_{max}$ and the table below results:

% Range Decrease: Range - (% Range Decrease) x Range = New Range
i.e., for a 12 dB sensitivity decrease, 500 miles - 50% x 500 miles = 250 miles

Range Multiplier: Range x Range Multiplier = New Range i.e., for a 12 dB sensitivity decrease 500 miles x 0.5 = 250 miles

Table 3. Effects of Sensitivity Decrease

dB Sensitivity Decrease	% Range Decrease	Range Multiplier	dB Sensitivity Decrease	% Range Decrease	Range Multiplier
- 0.5	3	0.97	-10	44	0.56
- 1.0	6	0.94	- 11	47	0.53
- 1.5	8	0.92	- 12	50	0.50
- 2	11	0.89	- 13	53	0.47
- 3	16	0.84	- 14	55	0.45
- 4	21	0.79	- 15	58	0.42
- 5	25	0.75	- 16	60	0.40
- 6	29	0.71	- 17	62	0.38
- 7	33	0.67	- 18	65	0.35
- 8	37	0.63	- 19	67	0.33
- 9	40	0.60	- 20	68	0.32