JAMMING TO SIGNAL (J/S) RATIO - CONSTANT GAIN [LINEAR] JAMMING

**JAMMING TO SIGNAL (J/S) RATIO (MONOSTATIC)**

\[
\frac{J}{S} = \frac{G_{ja(Rx)} G_j G_{ja(Tx)} \lambda^2}{4\pi\sigma} = \frac{G_{ja(Rx)} G_j G_{ja(Tx)} c^2}{4\pi\sigma f^2} \quad \text{(ratio form)}
\]

- \(G_{ja(Rx)}\) = The Gain of the jammer receive antenna
- \(G_j\) = The gain of the jammer
- \(G_{ja(Tx)}\) = The Gain of the jammer transmit antenna

or:

\[
10\log \frac{J}{S} = 10\log G_{ja(Rx)} + 10\log G_j + 10\log G_{ja(Tx)} - 10\log (4\pi\sigma/\lambda^2)
\]

or if simplified radar equations developed in previous sections are used:

\[
10\log \frac{J}{S} = 10\log G_{ja(Rx)} + 10\log G_j + 10\log G_{ja(Tx)} - G \quad \text{(dB)}
\]

* Keep \(\lambda\) and \(\sigma\) in same units. Note: \(\lambda = c/f\)

**JAMMING TO SIGNAL (J/S) RATIO (BISTATIC)**

Same as the monostatic case except \(G_{ja}\) will be different since RCS (\(\sigma\)) varies with aspect angle.

Since the jammer on the target is amplifying the received radar signal before transmitting it back to the radar, both \(J\) and \(S\) experience the two way range loss. Figure 1 shows that the range for both the signal and constant gain jamming have a slope that is 40 dB per decade. Once the jammer output reaches maximum power, that power is constant and the jamming slope changes to 20 dB per decade since it is only a function of one way space loss and the J/S equations for constant power (saturated) jamming must be used.

Normally the constant gain (linear) region of a repeater jammer occurs only at large distances from the radar and the constant power (saturated) region is reached rapidly as the target approaches the radar. When a constant gain jammer is involved it may be necessary to plot jamming twice - once using \(J\) from the constant power (saturated) equation [1] in Section 4-7 and once using the constant gain (linear) equation [4], as in the example shown in Figure 1.

**Figure 1.** Sample Constant Gain/Constant Power Graph

**K\(_2\) Values (dB):**

<table>
<thead>
<tr>
<th>RCS ((\sigma))</th>
<th>(f_1) in MHz</th>
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</tr>
</thead>
<tbody>
<tr>
<td>(units)</td>
<td>(K_2) =</td>
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CONSTANT GAIN SELF PROTECTION ECM

Most jammers have a constant power output - that is, they always transmit the maximum available power of the transmitter (excepting desired ECM modulation). Some jammers also have a constant gain (linear) region. Usually these are coherent repeaters that can amplify a low level radar signal to a power that is below the level that results in maximum available (saturated) power output. At some radar to target range, the input signal is sufficiently high that the full jammer gain would exceed the maximum available power and the jammer ceases to be constant gain and becomes constant power.

To calculate the power output of a constant gain jammer where:
- \( S_{Rj} \) = The Radar signal at the jammer input (receive antenna terminals)
- \( G_{Ja(Rx)} \) = The Gain of the jammer receive antenna
- \( G_j \) = The gain of the jammer
- \( \alpha_{Tx} \) = The one-way free space loss from the radar to the target
- \( P_{jCG} \) = The jammer constant gain power output
- \( P_j \) = The maximum jammer power output
- \( L_R \) = The jammer receiving line loss; combine with antenna gain \( G_{Ja(Rx)} \)

From equation [10], Section 4-3, calculate the radar power received by the jammer.

\[
10\log S_{Rj} = 10\log P_t + 10\log G_t - \alpha_{Tx} + 10\log G_{Ja(Rx)} \quad \text{(factors in } dB) \quad [1]
\]

The jammer constant gain power output is:

\[
10\log P_{jCG} = 10\log S_{Rj} + 10\log G_j \quad [2]
\]

and, by definition:

\[
P_{jCG} \leq P_j \quad [3]
\]

MONOSTATIC

The equivalent circuit shown in Figure 2 is different from the constant power equivalent circuit in Figure 4 in Section 4-7. With constant gain, the jamming signal experiences the gain of the jammer and its antennas plus the same space loss as the radar signal.

![Jammer Constant Gain ECM Equivalent Circuit (Monostatic)](image)

\( P_t \) = TRANSMITTER
\( G_t \) = RADAR ANTENNA GAIN
\( S_{Rj} \) = RECEIVER
\( P_r \) = (TOTAL SIGNAL = \( J + S \))
\( G_r \) = ONE-WAY SPACE LOSS
\( \alpha \) = ONE-WAY SPACE LOSS
\( \alpha_{1} \) = \( \alpha_{2} \) = \( \alpha_{J} \) = \( \alpha_{R} \)
\( G_{Ja(Rx)} \) = JAMMER RECEIVER ANTENNA GAIN
\( G_{Ja} \) = JAMMER TRANSMITTER ANTENNA GAIN
\( \alpha_{Tx} \) = ONE-WAY SPACE LOSS

For Monostatic: \( R_{Rx} = R_{Tx} \) \( \alpha_{Rx} = \alpha_{Jx} = \alpha_{Jx} = \alpha_{1} \)

Figure 2. Jammer Constant Gain ECM Equivalent Circuit (Monostatic)
To calculate $J$, the one way range equation from Section 4-3 is used twice:

$$J = \frac{P_t G_t G_{\text{ja}(\text{Rx})} \lambda^2}{(4\pi R)^2} - \frac{G_s G_{\text{ja}(\text{Tx})} G_r \lambda^2}{(4\pi R)^2}$$ \[4\]

From the two way range equation in Section 4-4:

$$S = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4}$$ \[5\]

Terms cancel when combined:

$$\frac{J}{S} = \frac{G_{\text{ja}(\text{Rx})} G_{\text{ja}(\text{Tx})} \lambda^2}{4\pi \sigma}$$ \[6\]

Keep $\lambda$ and $\sigma$ in the same units

Or in dB form:

$$10\log \frac{J}{S} = 10\log G_t + 10\log G_r + 10\log G_{\text{ja}(\text{Tx})} - 10\log \left(\frac{4\pi\sigma/\lambda^2}{8}\right)$$ \[7\]

Since the last term can be recognized as minus $G_\sigma$ from equation [10] in Section 4-4, where the target gain factor, $G_\sigma = 10\log (4\pi\sigma/\lambda^2) = 10\log (4\pi\sigma f^2/c^2)$, it follows that:

$$10\log \frac{J}{S} = 10\log G_{\text{ja}(\text{Rx})} + 10\log G_j + 10\log G_{\text{ja}(\text{Tx})} - G_\sigma$$ \[8\]

Target gain factor, $G_\sigma = 10\log \sigma + 20\log f_1 + K_2$ (in dB)

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BISTATIC

The bistatic equivalent circuit shown in Figure 3 is different from the monostatic equivalent circuit shown in Figure 2 in that the receiver is separately located from the transmitter, $R_{\text{Tx}} \neq R_{\text{Rx}}$ or $R_{\text{Jx}}$ and $G_\sigma$ will be different since the RCS (\sigma) varies with aspect angle.

**Figure 3.** Jammer Constant Gain ECM Equivalent Circuit (Bistatic)
To calculate \( J \), the one way range equation from Section 4-3 is used twice:

\[
J = \frac{P_i G_i G_{j\text{a}(Rx)}}{(4\pi R_{Tx})^2} \frac{G_j G_{j\text{a}(Tx)} G_r \lambda^2}{(4\pi R_{Rx})^2} \quad (R_{Jx} = R_{Rx}) \tag{9}
\]

From the two way range equation in Section 4-4:

\[
S = \frac{P_i G_i G_r \lambda^2 \sigma'}{(4\pi)^3 R_{Tx}^2 R_{Rx}^2} \quad (\sigma' \text{ is bistatic RCS}) \tag{10}
\]

Terms cancel when combined:

\[
\frac{J}{S} = \frac{G_{j\text{a}(Rx)} G_j G_{j\text{a}(Tx)}}{4\pi \sigma'} \quad \text{Keep } \lambda \text{ and } \sigma \text{ in same units} \tag{11}
\]

Or in dB form:

\[
10\log \frac{J}{S} = 10\log G_{j\text{a}(Rx)} + 10\log G_j + 10\log G_{j\text{a}(Tx)} - 10\log \left(\frac{4\pi \sigma' / \lambda^2}{4\pi \sigma'}\right) \tag{12}
\]

Since the last term can be recognized as minus \( G_\sigma \) from equation [10] in Section 4-4, where the target gain factor, \( G_\sigma = 10\log (4\pi \sigma' / \lambda^2) = 10\log (4\pi \sigma' f^2 / c^2) \), it follows that:

\[
10\log \frac{J}{S} = 10\log G_{j\text{a}(Rx)} + 10\log G_j + 10\log G_{j\text{a}(Tx)} - G_\sigma \quad \text{(factors in dB)} \tag{13}
\]

### Target gain factor, \( G_\sigma = 10\log \sigma + 20\log f_1 + K_2 \) (in dB)

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### Linear J/S (Monostatic) Example (Linear Power Jamming)

Assume that a 5 GHz radar has a 70 dBm signal fed through a 5 dB loss transmission line to an antenna that has 45 dB gain. An aircraft that is flying 31 km from the radar has an aft EW antenna with -1 dB gain and a 5 dB line loss to the EW receiver (there is an additional loss due to any antenna polarization mismatch but that loss will not be addressed in this problem). The received signal is fed to a jammer with a gain of 60 dB, feeding a 10 dB loss transmission line which is connected to an antenna with 5 dB gain.

If the RCS of the aircraft is 9 m^2, what is the J/S level received at the input to the receiver of the tracking radar?

Answer:

10\log J/S = 10\log G_{j\text{a}(Rx)} + 10\log G_j + 10\log G_{j\text{a}(Tx)} - G_\sigma

\[
G_\sigma = 10\log \sigma + 20\log f_1 + K_2 = 10\log 9 + 20\log 5 + 21.46 = 44.98 \text{ dB}
\]

Note: The respective transmission line losses will be combined with antenna gains, i.e. -1 -5 = -6 dB and -10 + 5 = -5 dB

10\log J/S = -6 + 60 - 5 - 44.98 = 4.02 \text{ dB @ 5 GHz}

The answer changes to 1.1 dB if the tracking radar operates at 7 GHz provided the antenna gains and aircraft RCS are the same at both 5 and 7 GHz.

\[
G_\sigma = 10\log 9 + 20\log 7 + 21.46 = 47.9 \text{ dB}
\]

10\log J/S = -6 + 60 - 5 - 47.9 = 1.1 \text{ dB @ 7 GHz}

Separate \( J \) (-73.5 \text{ dBm @ 5 GHz and } -79.34 \text{ dBm @ 7 GHz}) and \( S \) (-77.52 \text{ dBm @ 5 GHz and } -80.44 \text{ dBm @ 7 GHz}) calculations for this problem are provided in Sections 4-3 and 4-4, respectively. A saturated gain version of this problem is provided in Section 4-7.